Advances in Simulations of Generic Black-Hole Binaries

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Abstract. We review some of the recent dramatic developments in the fully nonlinear simulation of generic, highly-precessing, black-hole binaries, and introduce a new approach for generating hybrid post-Newtonian / Numerical waveforms for these challenging systems.

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1. Introduction

The field of Numerical Relativity (NR) has progressed at a remarkable pace since the breakthroughs of 2005 [1, 2, 3] with the first successful fully non-linear dynamical numerical simulation of the inspiral, merger, and ringdown of an orbiting black-hole binary (BHB) system. In particular, the 'moving-punctures' approach, developed independently by the NR groups at NASA/GSFC and at RIT, has now become the most widely used method in the field and was successfully applied to evolve generic BHBs. This approach regularizes a singular term in space-time metric and allows the black holes (BHs) to move across the computational domain. Previous methods used special coordinate conditions that kept the black holes fixed in space, which introduced severe coordinate distortions that caused orbiting-black-hole-binary simulations to crash. Recently, the generalized harmonic approach method, first developed by Pretorius [1], has also been successfully applied to accurately evolve generic BHBs for tens of orbits with the use of pseudospectral codes [4, 5].

Since then, BHB physics has rapidly matured into a critical tool for gravitational wave (GW) data analysis and astrophysics. Recent developments include: studies of the orbital dynamics of spinning BHBs [6, 7, 8, 9, 10, 11, 12], calculations of recoil velocities from the merger of unequal mass BHBs [13, 14, 15], and the surprising discovery that very large recoils can be acquired by the remnant of the merger of two spinning BHs [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 9, 29, 30, 31], empirical models relating the final mass and spin of the remnant with the spins of the individual BHs [32, 33, 34, 35, 36, 37, 38, 39], and comparisons of waveforms and orbital dynamics of BHB inspirals with post-Newtonian (PN) predictions [40, 41, 42, 43, 44, 45, 46, 47].

One of the important applications of NR is the generation of waveform to assist GW astronomers in their search and analysis of GWs from the data collected by ground-based interferometers, such as LIGO [48] and VIRGO [49], and future space-based missions, such as LISA [50]. BHBs are particularly promising sources, with the final merger event producing a strong burst of GWs at a luminosity of $L_{GW} \sim 10^{22} L_\odot$‡, greater than the combined luminosity of all stars in the observable universe. The central goal of the field has been to develop the theoretical techniques, and perform the numerical simulations, needed to explore the highly-dynamical regions and thus generate GW signals from a representative sample of the full BHB parameter space. Accurate waveforms are important to extract physical information about the binary system, such as the masses of the components, BH spins, and orientation. With advanced LIGO scheduled to start taking data in 2014-2015, there is a great urgency to develop these techniques in short order. To achieve these goals, the numerical relativity and data analysis communities formed a large collaboration, known as NINJA, to generate, analyze, and develop matched filtering techniques for generic BHB waveforms. A wide range of currently available gravitational waveform signals were injected into a simulated data

‡ This luminosity estimate is independent of the binary mass and takes into account that $3-10\%$ of the total mass M of the binary is radiated over a time interval of $\sim 100M$ [39].

set, designed to mimic the response of the Initial LIGO and Virgo gravitational-wave detectors, and the efficiency of current search methods in detecting and measuring their parameters were successfully tested [51, 52]. The next step will be a more detailed study of the sensitivity of current search pipelines to BHB waveforms in real data.

In order to create effective templates for GW data analysis, we need to cover the 7-dimensional parameter space of possible BHB configurations, including arbitrary mass ratios (1d) $q = m_1/m_2$ and arbitrary orientation and magnitudes of the individual BH spins (6d), in an efficient way. There are two important challenges here. The first challenge is to adapt the numerical techniques developed for similar-mass, low-spin BHBs to tackle BHBs with extreme mass ratios, i.e. $q < 1/10$ (See Refs. [19, 53, 54]) and, independently, the highly-spinning regime. In the latter regime, the binaries will precess strongly during the final stages of inspiral and merger, leading to large recoils and modulations in the waveform. These two regions are numerically highly-demanding due to the high resolution required for accurate simulations. A second challenge is to efficiently generate the waveforms numerically. Ideally one would like to have a bank of templates with millions of waveforms, but the computational expense of each individual simulation makes this unrealistic.

At RIT, we have been particularly interested in studying spinning BHBs and the effects of spin on the orbital dynamics, waveforms, and remnant BHs. In 2006 the RIT group began a series of analyses of spinning BHBs, with the goal of evolving a truly generic binary. Our studies began with the 'orbital hangup' configurations [6], where the spins are aligned or counter-aligned with the orbital angular momentum, and display dramatic differences in the orbital dynamics, see Fig 1. In this study we were also able to provide strong evidence that the merger of two BHs will produce a submaximal remnant (i.e. cosmic censorship is obeyed). We then analyzed spin-orbit effects [7] and found that they were too weak near merger to force a binary to remain in a corotational state. Afterwards, we analyzed spin-precession and spin flips [8]. With this experience, we were able to begin evolving 'generic' binaries, that is, binaries with unequal and unaligned spins, and mass ratios differing from 1 : 1 [17]. Remarkably, we found for a generic binary that the gravitational recoil out of the orbital plane, which is a function of the in-plane spin, was potentially much larger than any in-plane recoil. In fact, the measured recoil for our 'generic' configuration was actually as large as the largest predicted in-plane recoil (which assumed maximal spins perpendicular to the orbital plane) [16, 30] (see Fig. 2 for a plot of the radiated power per unit solid angle for a 'generic' BHB). Based on these results, we were able to predict a recoil of thousands of km/s for equal-mass, equal and anti-aligned spins (with spins entirely in the orbital plane). Based on our suggested configuration, the authors of Ref [21] evolved a binary with a recoil of 2500 km/s. However, our prediction indicated that the recoil can vary sinusoidally with the angle that the spins make with respect to the initial linear momentum of each hole. After completing a study of these superkick configurations with various spin angles, we were able to show that the maximum recoil was, in fact, much closer to 4000 km/s. Later on, we evolved a set of challenging superkick configurations,

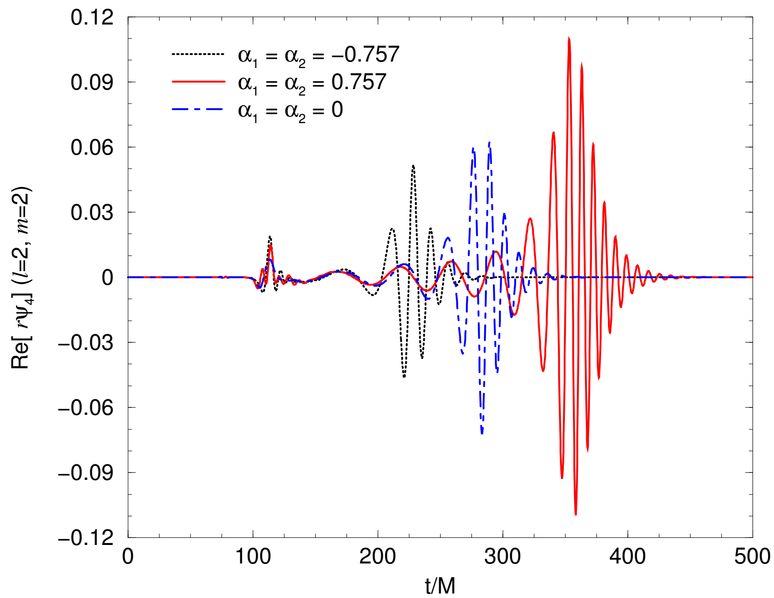

Figure 1. The hangup effect in the waveform for binaries with spins aligned and counter-aligned with the orbital angular momentum. In each case the binaries started out at the same orbital frequency.

with spins $S_i/m_H^2 = 0.92$, where m_H is the horizon mass, and found a recoil of 3300 km/s [55].

The cost of running a numerical simulation for many orbits, and in particular the cost of running a simulation with high spins and mass ratios that differ significantly from 1 : 1 means that we need to use hybrid analytic / numerical waveforms to model the full inspiral waveform. Combining post-Newtonian [56] (or Effective-one-Body [57]) waveforms and full numerical waveforms seems to be an ideal solution to this problem, but the modeling of even relatively distant (from an NR point of view) BHBs using PN is still an unsolved problem because the PN equations of motion are only known up to 3.5PN order, which as we show in Sec. 2 is not accurate enough to evolve close, highly-precessing, BHBs.

2. Inspiral and Merger of Generic Black-Hole Binaries

In [58], we compared the numerical relativity (NR) and post-Newtonian (PN) waveforms of a generic BHB, i.e., a binary with unequal masses and unequal, non-aligned, precessing spins. Comparisons of numerical simulations with post-Newtonian ones have several benefits aside from the theoretical verification of PN. From a practical point of view, one can directly propose a phenomenological description and thus make predictions in regions of the parameter space still not explored by numerical simulations. From the theoretical point of view, an important application is to have a calibration of the post-Newtonian error in the last stages of the binary merger.

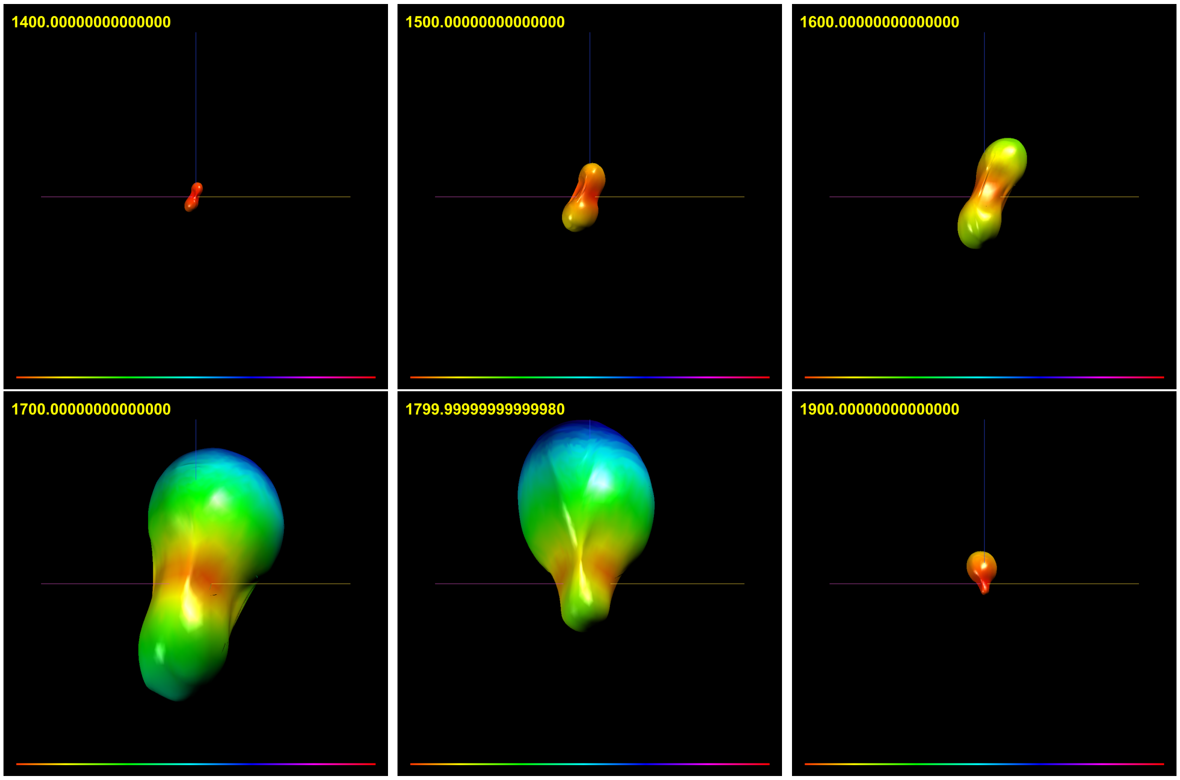

Figure 2. The radiated power per unit solid angle for a generic BHB configuration at different times. The asymmetry in the radiation leads to the instantaneous recoil.

To derive the PN gravitational waveforms, we start from the calculation for the orbital motion of binaries in the post-Newtonian approach. Here we use the ADM-TT gauge, which is the closest to our quasi-isotropic numerical initial data coordinates. We use the PN equations of motion (EOM) based on [59, 60, 61]. The Hamiltonian is given in [59], with the additional terms, i.e., the next-to-leading order gravitational spin-orbit and spin-spin couplings provided by [60, 61], and the radiation-reaction force given in [59]. The Hamiltonian we use here is given by

$$\begin{aligned} H = {} & H_{\mathrm{O,Newt}} + H_{\mathrm{O,1PN}} + H_{\mathrm{O,2PN}} + H_{\mathrm{O,3PN}} \\ & + H_{\mathrm{SO,1.5PN}} + H_{\mathrm{SO,2.5PN}} + H_{\mathrm{SS,2PN}} + H_{\mathrm{S_1S_2,3PN}} \,, \end{aligned} \tag{1}$$

where the subscript O, SO and SS denote the pure orbital (non-spinning) part, spin-orbit coupling and spin-spin coupling, respectively, and Newt, 1PN, 1.5PN, etc., refer to the perturbative order in the post-Newtonian approach. The $H_{\mathrm{S_1S_1(S_2S_2),3PN}}$ component of the Hamiltonian was recently derived in [62]. We should note that Porto and Rothstein also derived higher-order spin-spin interactions using effective field theory techniques [63, 64, 65, 66]. We obtain the conservative part of the orbital and spin EOMs from this Hamiltonian using the standard techniques of the Hamiltonian formulation. For the dissipative part, we use the non-spinning radiation reaction results up to 3.5PN, as well as the leading spin-orbit and spin-spin coupling to the radiation reaction [59]. Although, not used here, higher-order corrections to the spin dependent radiation

reaction terms were derived in [67, 68, 69, 70] and can be applied to our method to improve the prediction for the BH trajectories (and hence the waveform). This PN evolution is used both to produce very low eccentricity orbital parameters at $r \approx 11M$ (the initial orbital separation for the NR simulations) from an initial orbital separation of $50M$, and to evolve the orbit from $r \approx 11M$. We use these same parameters at $r \approx 11M$ to generate the initial data for our NR simulations. The initial binary configuration at $r = 50M$ had the mass ratio $q = m_1/m_2 = 0.8$, $\vec{S}_1/m_1^2 = (-0.2, -0.14, 0.32)$, and $\vec{S}_2/m_2^2 = (-0.09, 0.48, 0.35)$.

We then construct a hybrid PN waveform from the orbital motion by using the following procedure. First we use the 1PN accurate waveforms derived by Wagoner and Will [71] (WW waveforms) for a generic orbit. By using these waveforms, we can introduce effects due to the black-hole spins, including the precession of the orbital plane. On the other hand, Blanchet *et al.* [72] recently obtained the 3PN waveforms (B waveforms) for non-spinning circular orbits. We combine these two waveforms to produce a hybrid PN waveform. We note that there are no significant gauge ambiguities arising from combining the WW and B waveforms in this way because at 1PN order the harmonic and ADM gauges are equivalent (and hence the WW waveforms are the same in the two gauges) and the B waveforms are given in terms of gauge invariant variables. Also, it should be noted that we calculate the spin contribution to the waveform through its effect on the orbital motion directly in the WW waveforms and indirectly in B waveforms through the inclination of the orbital plane.

For the NR simulations we calculate the Weyl scalar ψ_4 and then convert the (ℓ, m) modes of ψ_4 into (ℓ, m) modes of $h = h_+ - ih_\times$.

To compare PN and numerical waveforms, we need to determine the time translation δt between the numerical time and the corresponding point on the PN trajectory. That is to say, the time it takes for the signal to reach the extraction sphere ($r = 100M$ in our numerical simulation). We determine this by finding the time translation near $\delta t = 100M$ that maximizes the agreement of the early time waveforms in the $(\ell = 2, m = \pm 2)$, $(\ell = 2, m = \pm 1)$, and $(\ell = 3, m = \pm 3)$ simultaneously. We find $\delta t \sim 112M$, in good agreement with the expectation for our observer at $r = 100M$. Since our PN waveforms are given uniquely by a binary configuration, i.e., an actual location of the PN particle, we do not have any time shift or phase modification other than this retardation of the signal. Note that other methods which are not based on the particle locations, have freedom in choosing a phase factor.

To quantitatively compare the modes of the PN waveforms with the numerical waveforms we define the overlap, or matching criterion, for the real and imaginary parts of each mode as

$$M_{\ell m}^{\Re/\Im} = \frac{< h_{\ell m}^{\mathrm{Num},\Re/\Im}, h_{\ell m}^{\mathrm{PN},\Re/\Im} >}{\sqrt{< h_{\ell m}^{\mathrm{Num},\Re/\Im}, h_{\ell m}^{\mathrm{Num},\Re/\Im} >< h_{\ell m}^{\mathrm{PN},\Re/\Im}, h_{\ell m}^{\mathrm{PN},\Re/\Im} >}}\,, \tag{2}$$

where $h_{\ell m}^{\Re/\Im}$ are defined by the real and imaginary parts of the waveform mode $h_{\ell m}$, respectively, and the inner product is calculated by $< f, g >= \int_{t_1}^{t_2} f(t)g(t)dt$. Hence,

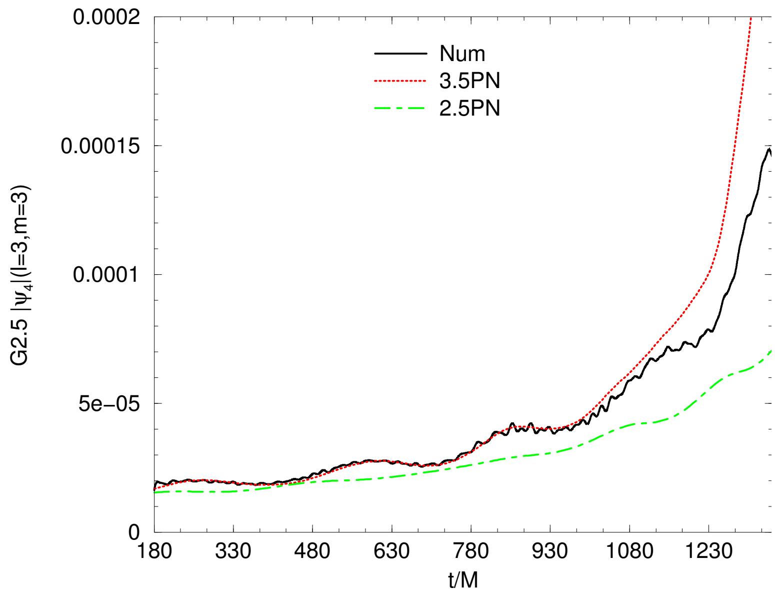

Figure 3. The amplitude of the $(\ell = 3, m = 3)$ mode of ψ_4 for the 'generic' binary configuration using the full numerical waveform, as well as waveforms derived from 2.5PN and 3.5PN EOMs. Note the much better agreement of the 3.5PN waveform, indicating that higher-order PN terms are important to the waveform during the late-inspiral phase.

$M_{\ell m}^{\Re/\Im} = 1$ indicates that the given PN and numerical mode agree. We analyzed the long-term generic waveform produced by the merger of unequal mass, unequal spins, precessing black holes, and found a good initial agreement of waveforms for the first six cycles, with overlaps of over 98% for the $(\ell = 2, m = \pm 2)$ modes, over 90% for the $(\ell = 2, m = \pm 1)$ modes, and over 90% for the $(\ell = 3, m = \pm 3)$ modes. The agreement degrades as we approach the more dynamical region of the late merger and plunge.

While our approach appears promising, there are some remaining issues. The PN gravitational waveforms used here does not include direct spin effects (spin contribution to the waveform arises only through its effect on the orbital motion). Recently, direct spin effects on the waveform were analyzed in [70].

In Fig. 3 we show the $(\ell = 3, m = 3)$ mode of ψ_4. A comparison of the PN and NR waveforms shows that there are significant errors in the 2.5PN approximate waveform that are significantly reduced by going to 3.5PN. However, it appears that still higher-order corrections are needed in order to accurately model the waveform using PN at an orbital radius of $r = 11M$. In Fig. 4 we show the orbital separation versus time. Here, as well, higher-order PN correction are important.

2.1. Hybrid Waveforms

To obtain a continuous and differentiable hybrid PN / NR waveform, we use a smoothing function to transition from the purely PN to purely NR parts of the waveform of the

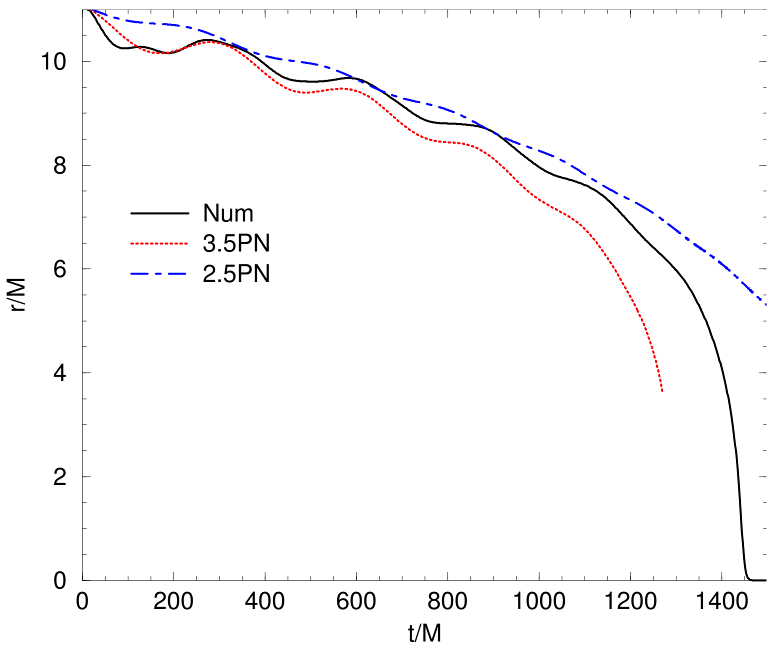

Figure 4. The orbital separation versus time the 'generic' binary configuration using the full numerical trajectories, as well as the trajectories derived from 2.5PN and 3.5PN EOMs. Note the much better agreement of the 3.5PN trajectories, and that 3.5PN captures the eccentricity of this configuration much better than 2.5PN, indicating that higher-order PN terms are important to orbital dynamics.

form

$$h = (1 - F(x))h^{PN} + F(x)h^{Num}\,, \tag{3}$$

where for example, we can use a simple polynomial,

$$F(x) = x^3(6x^2 - 15x + 10)\,. \tag{4}$$

This guarantees the C^2 behavior at $F(x) = 0$ and 1. In [73] the authors chose $F(x) = x$, which creates a discontinuity in the derivatives of the waveforms, especially ψ_4, and also an amplitude scaling factor to correct the amplitudes. Note that here we do not have any free parameters (we allow the time translation, here $\delta t \sim 112M$, to vary by $\sim 5\%$ about the retardation time ($T_{ret} \sim 109M$) of the observer location).

Figure 5 shows the hybrid waveform generated by the NR and PN waveforms for the binary discussed in the above section. Here, we use a half wavelength for the smoothing interval which starts at $t = 226.78875M$, and the time translation $\delta t = 112.64625M$ is considered.

3. Discussion

The remarkable progress in both analytic and fully non-linear numerical simulations of BHBs has made it possible to accurately model the inspiral waveform for a generic black-hole binary by combining both post-Newtonian waveforms from large separations

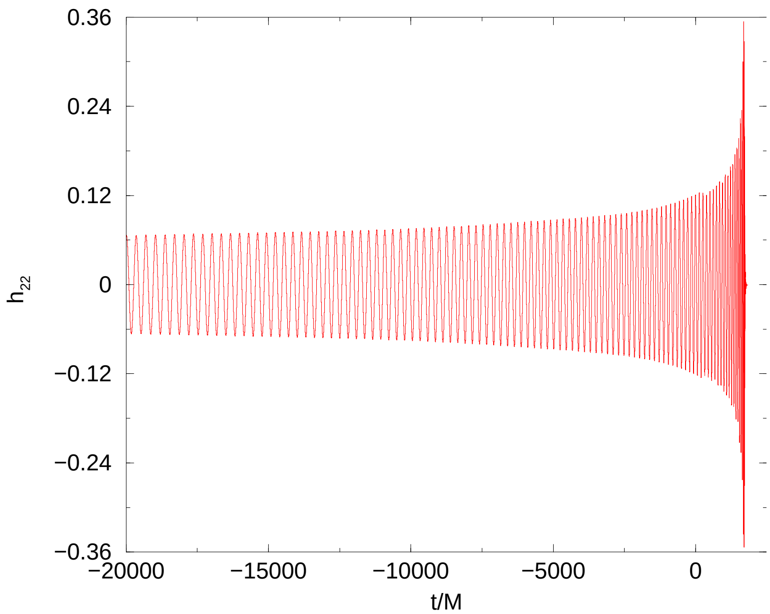

Figure 5. The real part of the $\ell = 2$, $m = 2$ mode of the hybrid waveform. This is created by matching the NR waveform to the waveform derived from 3.5PN EOMs.

and smoothly attaching this waveform to the corresponding fully non-linear waveform produced by the binary during the late-inspiral. We provide an example of one such hybrid waveform. Our waveform is available for download from `http://ccrg.rit.edu/downloads/waveforms`.

We found that 3.5PN produces a markedly better predicted waveform than 2.5PN, but there were still significant errors in the 3.5PN waveform for separations $r < 11M$. However, numerical simulations can start with larger separations (e.g. the 16-orbit simulation described in [4]) and there is significant progress in computing higher-order PN corrections. Hence, we expect that highly-accurate hybrid waveforms for generic binaries will soon be feasible.

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